3-D Parallel Finite Element Method with Prismatic Edge Elements for Electromagnetic Field Analysis of IPM Motor

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We developed a 3-D parallel finite element method with prismatic edge elements for electromagnetic field analysis of rotating machines. In this paper, the outline of the developed method is described. Moreover, the performance of the proposed method running on a PC cluster is quantitatively clarified through the dynamic characteristics analysis of an interior permanent magnet motor.

***Index Terms*— parallel finite element method, rotating machines, dynamic characteristics analysis, prismatic edge elements**

I. Introduction

We have reported a 3-D parallel finite element method (FEM) using the domain decomposition method (DDM) with prismatic edge elements [1] is more usefulness than that with tetrahedral edge elements [2] through the comparison of a dynamic analysis of an interior permanent magnet (IPM) motor [3].

For the purpose of speed up of analyzing the magnetic field with eddy current by the method shown in [3], we applied the $\boldsymbol{A}$-ϕ method to the 3-D parallel FEM with prismatic edge elements. Furthermore, we made it possible to be coupled with the equations of the magnetic field and the electrical circuit.

In this paper, the proposed method is described, and we clarified the usefulness of the proposed method through the dynamic characteristics analysis of an IPM motor.

II. Analysis Method

A. Fundamental Equations

The fundamental equations of the magnetic field are given by the magnetic vector potential $\boldsymbol{A}$ and the electric scalar potential ϕ as follows:

$$\mathrm{rot}(\nu\ \mathrm{rot}\ \boldsymbol{A}) = \boldsymbol{J}_0 + \boldsymbol{J}_e + \nu_0\ \mathrm{rot}\ \boldsymbol{M} \tag{1}$$

$$\boldsymbol{J}_e = -\sigma\left(\frac{\partial \boldsymbol{A}}{\partial t} + \mathrm{grad}\phi\right) \tag{2}$$

$$\mathrm{div}\,\boldsymbol{J}_e = 0 \tag{3}$$

where ν is the reluctivity, $\boldsymbol{J}_0$ is the exiting current density, $\boldsymbol{J}_e$ is the eddy current density, ν_0 is the reluctivity of the vacuum, $\boldsymbol{M}$ is the magnetization of permanent magnet, σ is the electric conductivity.

In order to analyze the electromagnetic field coupled with the electrical circuit, a set of (1)-(3) and the following voltage equations are solved simultaneously.

$$E = V_0 - RI_0 - \frac{d\Psi}{dt} = 0 \tag{4}$$

$$\Psi = \frac{n_c}{S_c}\int \boldsymbol{A}\cdot\boldsymbol{n}_s\, dv \tag{5}$$

$$\boldsymbol{J}_0 = \frac{n_c}{S_c} I_0\, \boldsymbol{n}_s \tag{6}$$

where V_0 is the applied voltage, R is the resistance, I_0 is the current and Ψ is the interlinkage flux of the coil, n_c and S_c are the number of turns and the cross-sectional area of the winding, respectively, and $\boldsymbol{n}_s$ is the unit vector along with the direction of exciting current.

B. Details of matrix equation in consideration of voltage source

The DDM is adopted for the parallel computing. Using the DDM, the analyzed domain is divided into multiple subdomains. In this case, the matrix equation in the i-th subdomain is given as follows:

$$\begin{bmatrix} \left[\dfrac{\partial G^{(i)}}{\partial A^{(i)}}\right] & \left[\dfrac{\partial G^{(i)}}{\partial \phi^{(i)}}\right] & \left[\dfrac{\partial G^{(i)}}{\partial I_0}\right] \\ \left[\dfrac{\partial G_d^{(i)}}{\partial A^{(i)}}\right] & \left[\dfrac{\partial G_d^{(i)}}{\partial \phi^{(i)}}\right] & [0] \\ \left[\dfrac{\partial E}{\partial A^{(i)}}\right] & [0] & \left[\dfrac{\partial E}{\partial I_0}\right] \end{bmatrix} \begin{Bmatrix} \{\delta A^{(i)}\} \\ \{\delta\phi^{(i)}\} \\ \{\delta I_0\} \end{Bmatrix} = -\begin{Bmatrix} \{G^{(i)}\} \\ \{G_d^{(i)}\} \\ \{V_0\} + \sum\limits_{\substack{j=1 \\ (j\neq i)}}^{n} \{F^{(j)}\} \end{Bmatrix} \tag{7}$$

where n is the number of subdomains, $\{G^{(i)}\}$ and $\{G_d^{(i)}\}$ are residuals obtained from (1)-(3) with the Galerkin method, E is the voltage formula in (4), V_0 is the applied voltage, and $\{F^{(j)}\}$ is the matrix-vector products which is evaluated in the j-th subdomain contains coil, and that is given as follows:

$$\{F^{(j)}\} = \begin{cases} \left[\dfrac{\partial E}{\partial A^{(j)}}\right]\{\delta A^{(j)}\} & \text{(if } j\text{-th subdomain contains coil)} \\ 0 & \text{(otherwise)} \end{cases} \tag{8}$$

The values of $[\partial E/\partial I_0]$ and $\{\delta I_0\}$ in (7) are the same in all subdomains.

C. Domain Decomposition Method for $\boldsymbol{A}$-ϕ Method

Fig. 1 shows an example of the DDM for $\boldsymbol{A}$-ϕ method. Fig. 1(a) shows the whole domain divided by the prismatic edge elements. It is desired that the analyzed region is divided into several subdomains while keeping the computation load evenly. Fig. 1(b) shows the subdomains when the whole domain is divided into two subdomains. From this figure, we can see that the number of edges in each subdomain is almost the same. The overlapped elements, which are as gray elements in Fig. 1(c), are given to each subdomain to exclude the data communications between subdomains in the assembling of the element coefficient matrix. Fig. 1(c) also shows the edges and nodes for data communications. It is possible to perform appropriate data communication in this process of matrix solution by using this relationship. We will describe the details in the full paper.

III. Numerical Results and Discussion

A. Analyzed Model and Analysis Conditions

Fig. 2 shows the analyzed model of an IPM motor. The analyzed region is 1/4 of the whole region because of the symmetry. The analysis is carried out taking into account the eddy current in the permanent magnet with a PC cluster, which consists of 16 PCs with Intel Xeon (3.4GHz). Table I shows the discretization data.

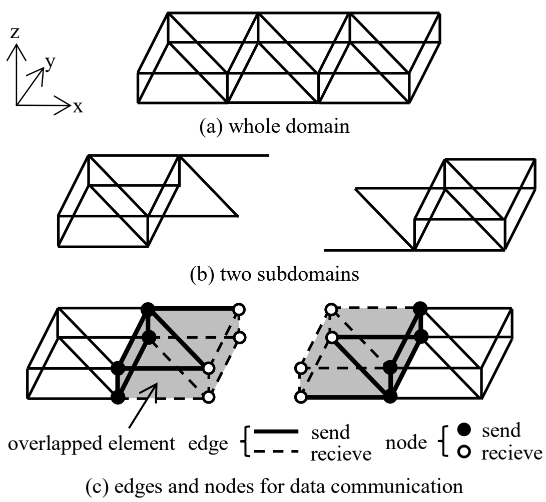

Fig. 1. Domain decomposition for $\boldsymbol{A}$-ϕ method with prismatic edge elements.

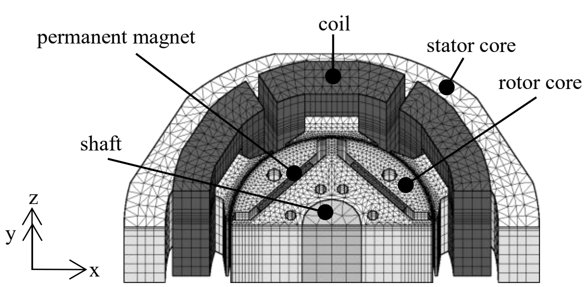

Fig. 2. Analyzed model of IPM motor.

TABLE I
DISCRETIZATION DATA

Number of elements	133,110
Number of Nodes	71,406
Number of edges	279,767
Number of unknown variables	254,756

B. Results Calculated and Discussion

Fig. 3 shows the calculated current waveforms of phase U. The waveform calculated by 16 PCs is almost the same as that calculated by one PC.

Fig. 4 shows the torque waveforms. The torque waveform calculated by 16 PCs is almost the same as that calculated by one PC.

Fig. 5 shows the distribution of eddy current loss in permanent magnet. The distribution agrees completely with that calculated by one PC.

Fig. 6 shows the speedup. From this figure, we can see that the speedup becomes large as the number of PCs increase. The speedup is 5.6 when the number of PC is 16.

In the full paper, the proposed method is described in detail. We also compared with the result of the voltage sources and current sources.

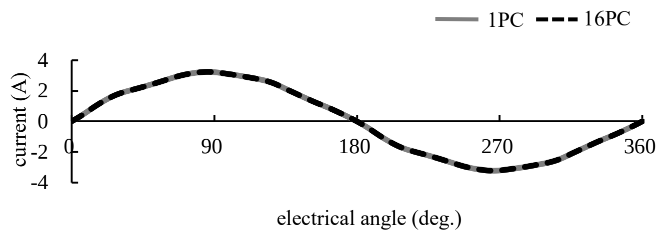

Fig. 3. Current waveforms (U phase)

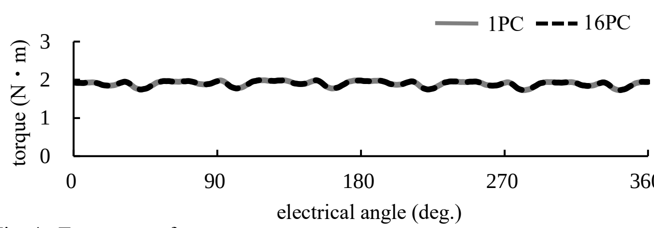

Fig. 4. Torque waveforms.

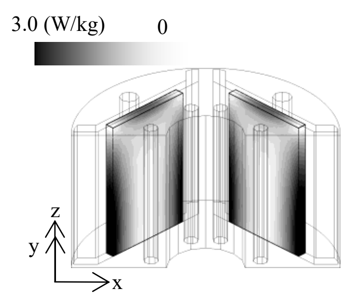

Fig. 5. Distribution of eddy current loss in permanent magnet.

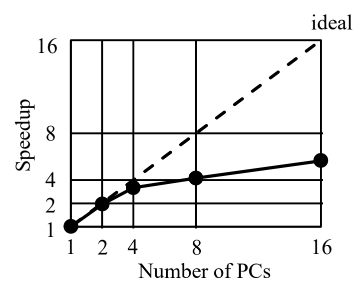

Fig. 6. Speedup.

References

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